APPROACH TO THE THEME PROBABILITY AND GEOMETRIC PROBABILITY IN HIGH SCHOOL MATHEMATICS BOOKS INDICATED BY PNLD

ABORDAGEM DO TEMA PROBABILIDADE E PROBABILIDADE GEOMÉTRICA NOS LIVROS DIDÁTICOS DE MATEMÁTICA DO ENSINO MÉDIO INDICADOS PELO PNLD

ENFOQUE DEL TEMA PROBABILIDAD Y LA PROBABILIDAD GEOMÉTRICA EN LOS LIBROS DIDÁCTICOS DE MATEMÁTICAS DE LA ESCUELA SECUNDARIA INDICADOS POR EL PNLD

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ABSTRACT: This work aims to analyze how the content of Probability is approached and whether Geometric Probability is presented in PNLD 2015 Mathematics textbooks. Samples of High School Mathematics suggested by the Textbook Guide were analyzed. It was noticed that all collections seek to highlight the historical aspects of Probability and most present situations of its application. None of the collections addresses the concept of Geometric Probability and only two have exercises that require this knowledge in the resolution. It is believed that the analysis performed can contribute to the teaching process of Probability concepts, making their learning more effective.

KEYWORDS: Didatic books. Math. PNLD. Analysis.

RESUMO: *Este trabalho objetiva analisar como é abordado o conteúdo de Probabilidade e se a Probabilidade Geométrica é apresentada em livros didáticos de Matemática do PNLD 2015. Foram analisados exemplares de Matemática do Ensino Médio sugeridos pelo Guia de Livros Didáticos. Percebeu-se que todas as coleções procuram evidenciar os aspectos históricos da Probabilidade e a maioria apresenta situações de sua aplicação. Nenhuma das coleções aborda o conceito de Probabilidade Geométrica e apenas duas apresentam exercícios que exigem esse conhecimento na resolução. Acredita-se que a análise realizada pode contribuir no processo de ensino dos conceitos de Probabilidade, tornando sua aprendizagem mais efetiva.*

PALAVRAS-CHAVE: *Livros didáticos. Matemática. PNLD. Análise.*

RESUMEN: *Este trabajo tiene como objetivo analizar cómo se aborda el contenido de Probabilidad y si la Probabilidad Geométrica se presenta en los libros didáticos de Matemáticas del PNLD 2015. Se analizaron muestras de Matemáticas de Secundaria sugeridas por la Guía de Libros Didáticos. Se notó que todas las colecciones buscan resaltar los aspectos históricos de la Probabilidad y las situaciones más presentes de su aplicación. Ninguna de las colecciones aborda el concepto de Probabilidad Geométrica y solo dos tienen ejercicios que requieren este conocimiento en la resolución. Se cree que el análisis realizado puede contribuir al proceso de enseñanza de los conceptos de Probabilidad, haciendo más efectivo su aprendizaje.*

PALABRAS CLAVE: *Libros didácticos. Matemáticas. PNLD. Analizar.*

Introduction

The knowledge of mathematics is important because they have applications in various areas of knowledge and in everyday life. Among this knowledge is probability that helps in the analysis of situations and decision making. Using Probability, you can determine the possibility of events occurring, such as the outcome of a lottery or the chances of a team winning a game or a championship.

The PCNEM (National Curriculum Parameters of High School) emphasize the importance of working mathematics in a contextualized way so that the student can analyze, understand, interpret, construct arguments and make decisions in everyday situations (BRASIL, 2000). Therefore, the curricula of basic education include several concepts of application of mathematics in daily life in its curriculum, as is the case of Probability. The study of this concept begins in Elementary School (EF) and is deepened in High School (EM).

Textbooks provided by the Ministry of Education (MEC) through the National Textbook Program (PNLD) are used as a reference in public schools. Schools choose the textbooks they wish to use through the analysis of the Textbook Guide that presents the review of the works approved by MEC. The objective of the PNLD is to make available to elementary and high schools, textbooks, collections of literary works, complementary works and dictionaries.

The textbook is an important material, because, according to Carvalho and Lima (2010) most teachers attribute to this resource a prominent role among the teaching resources that can be used in the teaching and learning process. Considering that the textbook is a material that is used by teachers as support in the teaching process, the importance of performing the analysis of this type of material is highlighted. Therefore, this work aims to analyze how the concept of Probability is approached and whether geometric probability is presented in the collections of mathematics textbooks indicated by the 2015 PNLD for High School. The analysis developed in this article is an integral part of the master's thesis entitled "The teaching of Geometric Probability: challenges and possibilities", authored by one of the authors, developed in the Graduate Program in Science and Mathematics Teaching of the Franciscan University.

This paper presents some reflections on the textbook and the teaching of mathematics; relevant aspects on the concepts of Probability and Geometric Probability; an overview of the works already published on this theme; the methodology used in this study; the approach of the theme Probability and Geometric Probability in high school mathematics textbooks; the final considerations and references used in the preparation of this work.

Textbook and mathematics teaching

The textbook is still an important didactic material for access to the content used by teachers in their classes and pointed out by Delizoicov, Angotti and Pernambuco (2007) as the main instrument of the teacher's work, in most classrooms, supporting teaching practice and serving as the main reference.

According to Carvalho and Lima (2010, p. 29) "A good textbook is a source for the knowledge of school mathematics. It is in him that we can become familiar with the mathematics that we must teach." The textbook can be used by the teacher as a support for the development of the content; this resource can also influence the way the teacher develops the content, and may propose situations problems or practical activities suggested in the textbook. Thus, the textbook signals which knowledge will be studied, the methods that can be adopted to learn it effectively and the curricular organization to the end of years of schooling (CARVALHO; LIMA, 2010). Textbooks generally aim to

> Provide students with a reference tool for any research; propose a partition of the program in structured chapters, allowing students and teachers to use them as a reference; be a database for exercises and problems; Explain a text, exposing the program's views (ALMOULOUD, 2005, p. 53, our translation).

The textbook plays the important role of favoring students' contact with school mathematics (CARVALHO; LIMA, 2010). The Common National Curriculum Base (BNCC) points out that the textbook is one of the resources that have an important role in the understanding and use of mathematical understanding and that "[...] these materials need to be integrated into situations that lead to reflection and systematization, in order to initiate a formalization process" (BRASIL, 2018, p. 276, our translation). Thus, the important role of the textbook is perceived and the relevance of performing analyses of this type of material is emphasized, aiming to identify its characteristics, strengths and aspects that need greater reflections. Textbooks

> [...] they are an important instrument for teaching and learning, it would be desirable for them to offer subsidies so that the teacher could develop his activities in an appropriate way to the official proposals, both in the preparation of his classes and in his teaching practice (NASCIMENTO, 2004, p. 51, our translation).

Carvalho and Lima (2010) emphasize that it is important to verify whether textbooks promote the integration between mathematical contents and relate mathematics to other areas

of knowledge. In addition, these authors also point out the importance of textbooks presenting suggestions for games, concrete materials, experiments, practical activities, investigations and projects.

In addition to these pedagogical aspects, textbooks are resources that favor research in any time and place; because it has printed versions and can be handled by its users, even without internet access.

In view of the aspects presented, it is perceived the importance that the textbook plays as a subsidy for the teacher to plan his classes and as a source of consultation for students. Thus, it is important to carry out studies that analyze this type of material, which can help teachers identify the potentialities of this resource.

Geometric Probability and Probability

Probabilities theory makes it possible to create models to study random experiments or phenomena. A random phenomenon is every experiment that, even being repeated several times, under the same conditions, does not present the same result; because of this is that if you seek the chances, the probable results, the probability of a given result occurring.

The first formal definition of probability as a quotient of the number of "favorable cases" on the number of "possible cases", according to Morgado *et al.* (1991), was found in *The Liber of Ludo Aleae* by Jerome Cardano, which is described below. Consider that random experiments have the following characteristics:

a) There is a finite number (called *by n*) of elementary events (possible cases). The sample space (Ω) is the union of all elementary events.

b) Elementary events are equiprobable (all have the same probability of occurring).

c) Every event *A* is a union of elementary events where *m n*◦ (favorable cases).

It is then defined:

$$\text{Probability of A} = P(A) = \frac{\text{number of favorable cases } m}{\text{number of possible cases}} = \frac{m}{n}$$

The knowledge of Probability is important, because it helps people in the analysis of situations and decision-making in their daily lives. In addition, it allows the student to develop important skills such as critical analysis and argumentation (LOPES, 2008).

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Woodward and Hoehn (1994) point out that in order for students to learn more about Probability, it is necessary to teach or reinforce concepts of this content relating to other contents such as Geometry. According to the authors, Geometry is appropriate for the following factors: geometry problems involving probability are interesting and can serve as motivation; students will be able to use differently the concepts of Geometry they already have; and will have a better understanding of the concepts of Probability when they see them applied to the context of geometry.

Geometric Probability helps students to construct important concepts, as it uses the concepts of Geometry studied since elementary school to develop the concept of Probability (FREITAS, 2009).

The content of Geometric Probability is not always addressed in Basic Education Mathematics classes (ALCÂNTARA, 2014; CAETANO; PATERLINI, 2013). This fact can happen because probability concepts are often not worked on at this level of education, although it is an important topic that is used in several areas of knowledge (SILVA, 2014; VIANA, 2013).

Geometric Probability is a part of the Probability study in which it is necessary to use geometric stakes to solve probabilistic problems (GONDIM, 2013; SILVA, 2014). Length, area, and volume are the most commonly used geometric nodes to solve this type of problem. According to Pereira (2011, p. 1, our translation): "[...] in geometric probability theory, random elements are not quantities, but geometric objects such as line points and rotations."

Tunala (1992, p.16, our translation) places that:

> Some probability problems are equivalent to random selection of points in sample spaces *represented by geometric figures*. In the models under this case, the probability of a given event is reduced to the relationship – or its limit, if any – between homogeneous *geometric measurements*, such as length, area or volume.

According to Machado (2012), Probability in geometric spaces are ratios between measures of length, area or volume, and these measurements are in the same unit. In this sense, Tunala (1992) presents the following definition for probability involving area: Suppose *that a flat figure is part* of another flat figure *A* and that a point of A has been chosen *at random*. If we admit that the probability of this point belonging to *a* is proportional to the area of *a* and does not depend on the place that occupies it in *A*, then the probability that the selected point is in *a* will be:

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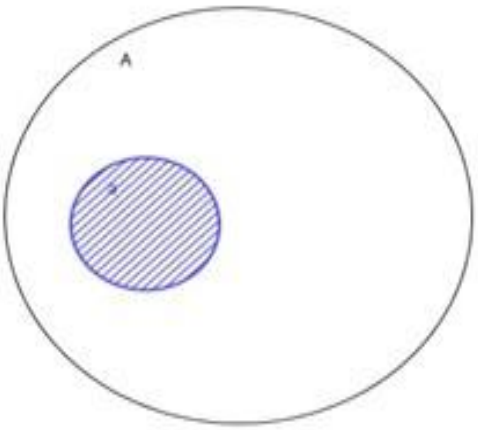
Figure 1 – Area probability ratio of two geometric spaces



Source: Elaborated by the authors (2020)

Probability in textbooks: overview

In this section, some studies are presented that analyzed how the concept of probability is addressed in mathematics textbooks.

Rodrigues and Martins (2016) aimed to analyze and discuss how the theme Probability is addressed in the six collections of textbooks approved for high school in the PNLD of the triennium 2015-2017. From the analysis of textbooks, the authors emphasize that probability content is not addressed in any of the collections analyzed in the first year of high school; two addresses in volumes 2 and 3, and the other present this content only in the second volume; thus, probability content is worked on in only one year of the high school cycle. The authors also found that collections usually dedicate, on average, only a little more than three percent of the pages to the approach of Probability content, throughout high school.

Viali and Oliveira (2010) interviewed high school teachers to identify which textbook collections they usually use. From the answers, the authors selected the five books most cited by the teachers for analysis. The objective of the analysis was to verify how the content of Probability is presented and whether it is in accordance with the legislation, specifically the PCN+. The authors found that probability content is usually inserted in the last chapter, without establishing relationships with previous contents and that in the examples and exercises, issues involving games, currencies and data prevail. Finally, they point out that none of the texts make use of or encourage the teacher to use technological resources.

Coutinho *et al.* (2013) conducted a study of the 2012 PNLD Textbook Guide in relation

to the presence of probability content, the number of pages destined to it and the methodology used. Then, the authors analyzed the tasks proposed in one of the collections, which was chosen because it had the least number of pages destined to Probability. The authors concluded that the analyzed work presented a good approach to probability content, which contemplated from the perception of chance, through the description of a random experiment, enumeration of possible results, and then addressed the classical definition of Probability. They also realized that the frequentist vision[1] is not adequately explored in the approved collections presented in the Textbook Guide.

Oliveira (2006), analyzed how the contents of Statistics and Probability are approached in a sample of mathematics textbooks for high school, published between 1992 and 2005. From the analysis, the author points out that textbooks give little emphasis to the contents of Probability and Statistics; according to the author, some present misconceptions, do not contextualize the themes and do not propose the use of information and communication technology (ICT) resources in problem solving, thus going against the Educational Guidelines complementary to national curriculum parameters (PCN+).

In view of the aspects highlighted in the studies found in the literature, the importance of performing textbook analysis is emphasized, since these have aspects that deserve the teacher's attention at the time of their use.

Methodology

The work presented is a qualitative study, of the documentary type. According to Lüdke and André (1986, p. 38, our translation), documentary analysis is an important qualitative data approach, which analyzes "[...] from laws and regulations, norms, opinions, letters, memos, personal diaries, autobiographies, newspapers, magazines, speeches, scripts for radio and television programs to books, statistics and school archives".

The objective of this work is to analyze how probability concepts are approached and whether geometric probability is presented in high school mathematics textbooks of the 2015 PNLD. To this end, all the copies suggested by the 2015 PNLD Textbook Guide, destined to high school (Chart 1) were performed. The Textbooks Guide presents the contents addressed in each volume of textbooks of all collections, thus, from their analysis and review of the volumes

[1] Probability Regular is characterized by the mathematical value of probability being resulting from the experimentation process, in which the frequency with which the facts occur is observed.

of the collections presented in Chart 1, we select the volumes that address the concept of Probability.

Table 1 – Reference of the collections of High School books analyzed

COLLECTIONS	REFERENCE
Collection I	LEONARDO, F. M. **Conexões com a Matemática**. Vol. 2, 3. 2ª ed. Cidade: Moderna, 2013.
Collection J	DANTE, L. R. **Matemática:** Contexto e Aplicações. Vol. 2, 3. 2ª ed. São Paulo: Ática, 2013.
Collection K	PAIVA, M. R. **Matemática – Paiva**. Vol. 2. 2ª ed. Cidade: Moderna, 2013.
Collection L	IEZZI, G.; DOLCE, O.; DEGENSZAJN, D. M.; PÉRIGO, R.; ALMEIDA, N. S. **Matemática** – Ciência e Aplicações. Vol. 2. 7ª ed. São Paulo: Saraiva, 2013.
Collection M	SMOLE, K. C. S.; DINIZ, M. I. S. V. **Matemática** – Ensino Médio. Vol. 2, 3. 8ª ed. São Paulo: Saraiva, 2013.
Collection N	SOUZA, J. R. **Novo olhar:** Matemática. Vol. 2. 2ª ed. São Paulo: FTD, 2013.

Source: Ritter (2017, p. 61)

The analysis performed in the collections of textbooks was based on the following questions: How is the content of Probability addressed? Are the historical aspects of how this content emerged? Are content application situations displayed? Is Probability related to other math content? Is Probability related to other disciplines/areas? Are activities proposed such as experiments, construction of conceptual maps, games, among others? Is the concept of Geometric Probability addressed? Are exercises on Geometric Probability proposed? These questions were proposed based on the points that Carvalho and Lima (2010) consider important to be observed in mathematics textbooks and on the aspects that are important in the approach of this content.

Approach to geometric probability and probability in high school textbooks

It is noteworthy that the analysis of the textbooks presented below is an integral part of the dissertation "The teaching of Geometric Probability: challenges and possibilities" of one of the authors of this work. The aim of this study is to analyze how probability content is approached and whether Geometric Probability is presented in 2015 PNLD High School mathematics textbooks.

By analyzing Chart 1, it is perceived that half of the analyzed collections (I, J and M) approach probability content in two volumes while the others (K, L and N) approach in only

one. A brief description of the collections analyzed is presented below.

Leonardo's collection I (2013) shows how the theory of probabilities emerged, also presents applications of this theory in scientific research and in social life. In this collection, the concepts are exposed in an interspersed way with examples and applications of Probability are presented in everyday situations. Some resolved exercises and questions about them are proposed for students to reflect and discuss. In the section "Complementary exercises" are presented exercises that are classified as: "Application, Deepening, Challenge, Self-Assessment", some of which are vestibular and ENEM. The collection has the section "Text comprehension" in which it is suggested that an interdisciplinary work with Biology be carried out, addressing for this theme blood transfusion; the collection highlights the contributions of various sciences in this process, including Mathematics. A discussion is presented on this topic and then some questions about it are proposed and involving Probability. Geometric Probability is not worked or exercises on this subject are proposed.

Dante (2013), collection J, expose how the study of probability theory arose. In this collection, the concepts are developed in an interspersed way with exercises and resolved exercises. Some of the resolved exercises have the topic "Solved step by step" in which a sequence of steps is performed that are: "Reading and Understanding," "Planning the Solution," "Running What Was Planned," "Emitting the Response," and "Extending the Problem." The collection also presents "Applications of Probability to Genetics" and points out that this area is perhaps the branch of Biology that most utilizes the mathematical concepts of probability theory. In this section, exercises that address this theme are also presented. The section "Other contexts" discusses organ donation, and probability issues that address this theme. It presents the section "Readings" which has the topics "The Mathematics of Luck" and "A Little More about Probabilities" discussing these subjects. Geometric Probability is not worked on nor are exercises proposed on this subject in this collection.

Paiva's Collection K (2013) presents a brief historical account of the emergence of probability theory. The concepts are presented and exemplified, and then "Resolved Exercises" and "Proposed Exercises" are presented. The exercises are contextualized with real situations.

It presents a section called "Work Roadmap", in which topics related to the concepts worked for students to discuss in groups are presented. They propose "Complementary exercises", some of these vestibular and ENEM. They also propose the "Resolution Analysis", in which an exercise is presented and its resolution, during which a mistake was made, then encourages the identification of this error and to remake the resolution. The section

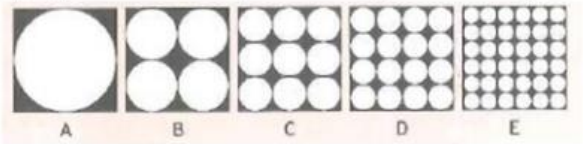
"Mathematics without borders" is also presented, which addresses the "Life Expectancy" of the Brazilian population, and probability questions are proposed about the data presented. Geometric Probability is not worked or exercises on this subject are proposed.

Iezzi *et al.* (2013), collection L, address the history of Probability in the section "A little history" in which they present the first records linked to the theory of Probability. The concepts are presented together with real situations and questions are proposed that use for their resolution the ideas of combinatorial and Probability. The exercises are contextualized with real situations, and several situations are presented in which the concepts of Probability are used, such as the probability of winning in the lottery. Geometric Probability is not explored in this collection.

The collection M, by Smole and Diniz (2013), in volume 2, highlights some areas of probability applications. The contents are explained and exemplified. In the "Problems and exercises" section, it suggests conducting experiments such as coin and data releases and contextualized problem solving. They also propose exercises and their respective resolution, encouraging the student to analyze the resolution to identify if it has errors, encouraging them to argue about them. It has the "Invent you" section, which suggests that students create problems on a particular subject. It presents links from digital games as a suggestion that can be used to help in understanding the concept of Probability. It has solved exercises; in the "To learn more" section are proposed activities such as the posting of coins. It also addresses the relationship between Probability and Counting, and exercises on these contents are proposed. One of the exercises (Chart 2) proposed in the "Get out of this" section requires the notion of Geometric Probability for resolution.

Table 2 – Problem proposed in book M

2) Figures A to E represent targets from a game.



Points in this game are scored according to the rules

1 point for those who throw the dart into the circle.

No point for those who throw the dart inside the square, but outside the circle.

If the dart is thrown outside the square, you can throw it again.
In these conditions, which of the 5 targets would you choose to play? How do you justify your choice, using Math. _____

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Source: Smole and Diniz (2013, p. 181)

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It has the section "Project Probability: history and applications" in which it suggests that students investigate the history of Probability and its applications. In the "keywords" section, they propose the construction of conceptual maps as a way to synthesize what has been studied. To this do, it shows an initial outline of a conceptual map on the topic of Probability and suggests that students complete it.

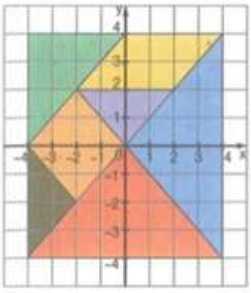
It presents the section "Mathematical Connection - Quantum Physics", which has a text on the relationship between Quantum Theory and Probability.

In volume 3 of this collection, the unit "Probability and Statistics" is presented, which aims to use Probability in the interpretation of situations and in decision-making based on statistical data. It has two sections of deepening of the contents entitled "Function or distribution of Probability" and "Frequent probability and law of large numbers". In the "On the computer" section, they suggest a computer program that makes simulations such as the launch of a coin; presents instructions for using the program, urging students to explore it and conduct experiments. In the section "To know", a text about a family of mathematicians is presented that contributed to the theory of Probabilities. In the section "Mathematical Connection - Forensic Science - Probability" is presented a text on Forensic Science and the various areas involved for the scientific analysis of the evidence of a crime.

Souza (2013), collection N, presents the origin of probability theory related to gambling. The concepts are exposed interspersed with examples. In the "Resolved Activities" section, some exercises are presented along with their resolution. It also presents the "Activities" section in which exercises are proposed for students to solve. Both in the "Resolved Activities" and in the "Activities" the book presents questions of the ENEM and vestibular. In the "Activities" section one of the proposed questions is about Tangram. This exercise does not mention Geometric Probability, but for its resolution it is necessary to know this concept, as can be observed in Chart 3.

Table 3 – Problem 1 proposed in book N

Tangram is an ancient Chinese game, made of seven geometrical forms that, when organized, can form 1700 silhouettes (silhouette: an image that represents an object or a person according to their shadow). The Chinese call it "Wisdom table" or "Seven wisdoms table". Consider the Tangram on a Cartesian graphic.



a) What is the probability that, when we randomly mark a point belonging to the Tangram, that point

- Belongs to an orange region?

- Belongs to a purple region?

- Doesn't belong to a blue region?

b) If we mark a point belonging to the Tangram with abscissa -3, what is the probability that this point belongs to the red region? What about the green region?

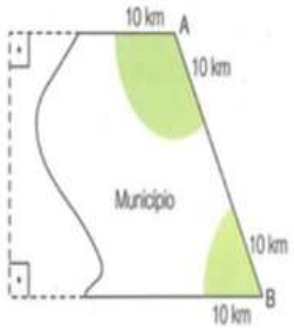
c) If we mark a point belonging to the Tangram with positive ordinate, what is the probability that this point belongs to the yellow region?

Source: Souza (2013, p. 257)

It has the "Context" section that covers several topics such as probability in a group of people, two of whom have a birthday on the same day. It presents a text that deals with the contributions of Probability in the diagnosis of diseases in medicine and then proposes some questions. One of the exercises proposed in the section "Complementary Activities" of the ENEM (Chart 4) also requires knowledge of Geometric Probability for its resolution.

Table 4 – Problem 2 proposed in book N

A 628km2 city has 2 radio stations. Both antennas A and B can reach a 10km distance, as shown in the figure.



To hire an advertising contract, an agency needs to evaluate the probability that a resident has of circulating freely in the municipality, being within the reach of at least one of the broadcasters.

This probability is approx.

a)20% b) 25% c) 30% d) 35% e) 40%

Source: Souza (2013, p. 276)

In the section "Exploring the theme" presents "The problem of the five discs" according to the version of Malba Tahan. They also propose a probabilistic analysis of their solution. Table 5 presents a synthesis of the most evident aspects observed and that answer the questions that we propose to analyze in the collections of mathematics textbooks.

Table 5 – More evident aspects of probability content in the analyzed collections

Aspects/Collection	**I**	**J**	**K**	**L**	**M**	**N**
It addresses its emergence.	X	X	X	X	X	X
Relates to other content.				X	X	
Relates to other disciplines/area.	X	X	X		X	X
They propose applications to everyday situations.	X	X	X	X	X	X

It suggests the performance of activities (experiments, conceptual maps, games).					X	
It has exercises that involve Geometric Probability.					X	X

Source: Prepared by the authors (2021)

Regarding how the contents are presented in the six collections of textbooks analyzed, we can perceive that all of them seek to use a story approach of how the first senses of Probability emerged. Another relevant aspect observed is that all collections present situations of application of probability content to everyday situations. These data are in line with what Carvalho and Lima (2010) states, that most textbooks seek to relate mathematical contents to their meaning in people's daily lives.

Vestibular (application exams for college) and ENEM exercises were present in three of the six collections of textbooks analyzed, which is also a relevant aspect, since this type of exercise helps to familiarize the student with the type of questions of these selection processes.

It was also noticed the concern of the majority (5) of the collections in relating the content of Probability to other disciplines/areas, this aspect is important because it is a content that is used in several disciplines, such as Biology. It is also notelike the concern of the collections in presenting examples of how probability can be used in other contexts, such as: in the diagnosis of diseases in medicine, in organ donation, in the life expectancy of the Brazilian population and in Forensic Science.

It is noteworthy that only in a collection, collection M, activities such as: experiments, digital games to be played by students are suggested; construction of conceptual maps. Thus, it is perceived that this collection seeks to stimulate the use of different resources and strategies to contribute to the teaching and learning process of probability concepts.

Another aspect that needs to be further explored in the analyzed collections is to relate probability content with other mathematics contents, such as combinatorial analysis, for example, since only two collections (L and M) present these relationships.

The analysis also made it possible to verify that none of the textbook collections addresses the concept of Geometric Probability; two present exercises that for their resolution it is necessary to use this concept. Lima (2001), in his study performed an analysis of twelve

collections of mathematics textbooks used in the three years of high school in Brazil, finding the content of Geometric Probability in only one of them. This author points out that "This subject, rarely addressed in high school, is an appropriate introduction to the notion of continuous probability and provides very motivating examples" (LIMA, 2001, p. 97). According to Caetano and Paterlini (2013) the concept of Geometric Probability is little worked on in high school, but their study is important, as it can help students associate the concepts worked in Probability with the geometric knowledge they already have.

Ritter and Bulegon (2017) performed an analysis of how the content of Geometric Probability and Probability is worked in the elementary school mathematics textbooks indicated by the PNLD 2014. These authors found that in some collections the concept of Geometric Probability is mentioned and some exercises are proposed on it. Thus, it is perceived that the concept of Geometric Probability is addressed in some textbooks of Elementary School Mathematics indicated by PNLD 2014, while in the high school mathematics textbooks indicated by PNLD 2015, this concept appears only intuitively in a few exercises.

Final considerations

The objective of this work was to analyze how probability content is approached and whether geometric probability is presented in 2015 PNLD mathematics textbooks for high school students. The analysis of the six collections of textbooks made it possible to realize that the concept of Geometric Probability is not addressed in any, but in two of them exercises are proposed that, for its resolution, it is necessary to use this concept.

It can be concluded that the collections analyzed use different approaches to present the content of Probability, and it was possible to identify the prevalence by using the historical approach of how this content arose and the concern to present everyday situations of its application.

Only one of the collections analyzed suggested the use of resources, such as games, in the teaching of Probability. Thus, it is important that collections incorporate suggestions of practical activities using different resources and strategies, since this type of activity has great potential to contribute to the teaching and learning process of various concepts.

Moreover, it is noteworthy that as pointed out by Carvalho and Lima (2010) the textbook is an important resource, but this should not be the only resource used to support the teacher's work, it is necessary to seek other sources to expand and improve the content and adapt it to the

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reality of the student group.

Finally, it is emphasized that in 2020/2021, with the Covid-19 pandemic and difficulties in accessing the Internet, many students only had contact with the concepts, studied in Basic Education, through textbooks. Thus, it is believed that the textbook is an important material and that the analysis performed can contribute to the teaching and learning process of the concepts of Probability, making its learning more effective.

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